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AN INVESTIGATION OF ESTIMATION TECHNIQUES

IN OPTIMALLY GUIDED AIR-TO-AIR MISSILES

by

Royce D. Harbor

PREFACE

The work described herein involves an optimal guidance system
for an air-to-air missile, developed through the Systems Analysis
and Simulation Branch of the Air Force Armament Laboratory, Eglin
Air Force Base, Florida. The work extends from that accomplished
by the author under the AFOSR Summer Faculty Research Program
during the summer of 1980 (Contract No. F49620-79-C-0038). In
this summer work, an investigation was made for the purpose of
explaining the fact that certain states estimated by an extended
Kalman filter differed significantly from the same quantities
generated by a truth model. Results indicated that the probable
cause for this behavior was the state model of the system used
in the extended Kalman filter. In the work described herein,
the study was extended to include the treatment of differences
observed in system behavior when under the influence of extended
Kalman filters based upon six-state and nine-state models.

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I. INTRODUCTION:

In optimizing a feedback control system relative to some performance index, the optimal control law can require the feedback of system states which are not readily measured. When this is the case, an effort is usually made to generate the inaccessible states from those quantities which can be measured. Frequently such measurements are corrupted by noise. In such cases, if the system is linear, the computational algorithm known as a Kalman filter provides a means for generating the needed state variables from measured variables corrupted by noise. If the state equations and/or the measurement equations are nonlinear, nonlinear filtering techniques are required. In such cases a satisfactory solution is often found in the extended Kalman filter.

The system being considered in this work is a short-range air-to-air missile in which an optimal control law is implemented for the purpose of minimizing miss distance. Quantities measured are azimuth and elevation angles of the target relative to the missile position. The measurement equation is nonlinear and is given by[1]

$$z(t_k) = g(x(t_k)) + v(t_k) \tag{1}$$

where

$$g(t_k) = \left\{ \begin{array}{c} \tan^{-1}\left[-z_R^2/(x_R^2 + y_R^2)^{\frac{1}{2}} \right] \\ \tan^{-1}(y_R/x_R) \end{array} \right\}_{t_k} \tag{2}$$

where x_R, y_R, and z_R are relative position components in inertial

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coordinates, and $v(t_k)$ is measurement noise assumed to be zero mean with covariance R.

Two extended Kalman filters were considered in this study. One was based upon a six-state model of the system dynamics, and the other was based upon a nine-state model. For the nine-state model, the elements of the state vector are defined as follows:[1]

x_1: relative position in the x direction

x_2: relative position in the y direction

x_3: relative position in the z direction

x_4: relative velocity in the x direction

x_5: relative velocity in the y direction (3)

x_6: relative velocity in the z direction

x_7: target acceleration in the x direction

x_8: target acceleration in the y direction

x_9: target acceleration in the z direction

Here relative position (velocity) means target position (velocity) relative to missile position (velocity). The state equation is written

$$\dot{\underline{x}} = F\underline{x} + \underline{b} + \underline{w} \qquad (4)$$

where F is a constant matrix. Written as a partitioned matrix of three by three partitions, it can be represented as

$$
F = \begin{bmatrix} 0 & \vdots & I & \vdots & 0 \\ \hdashline 0 & \vdots & 0 & \vdots & I \\ \hdashline 0 & \vdots & 0 & \vdots & -\lambda_T \end{bmatrix} . \tag{5}
$$

The matrix λ_T represents a matrix of time constants

$$
\lambda_T = \begin{bmatrix} \lambda_{T_x} & 0 & 0 \\ 0 & \lambda_{T_y} & 0 \\ 0 & 0 & \lambda_{T_z} \end{bmatrix} . \tag{6}
$$

The vector $\underline{b}$ consists of missile accelerations

$$
\underline{b} = \begin{bmatrix} 0 & \vdots & -\underline{a}_M^T & \vdots & 0 \end{bmatrix}^T \tag{7}
$$

where

$$
\underline{a}_M = \begin{bmatrix} a_{M_x} & a_{M_y} & a_{M_z} \end{bmatrix}^T . \tag{8}
$$

The vector $\underline{w}$ embodies noise in the missile and target acceleration, and is written

$$
\underline{w} = \begin{bmatrix} 0 & \vdots & \underline{w}_M^T & \vdots & \underline{w}_T^T \end{bmatrix}^T \tag{9}
$$

where

$$
\underline{w}_M = \begin{bmatrix} w_{M_x} & w_{M_y} & w_{M_z} \end{bmatrix}^T \tag{10}
$$

ana

$$\underline{w}_T = \begin{bmatrix} w_{T_x} & w_{T_y} & w_{T_z} \end{bmatrix}^T .$$ (11)

The state model for the six-state filter follows in straight-forward fashion by diminishing the nine-state model. In this model, the state vector consists of the first six states of the nine-state filter.

A detailed digital computer simulation program for this system was utilized in this study. The program was designed and put into use by engineers at the Systems Analysis and Simulation Branch of the Air Force Armament Laboratory at Eglin Air Force Base. Exclusive of detail, the program package simulates the system in accordance with the non-detailed block diagram of Figure 1. It embodies both six and nine-state filters.
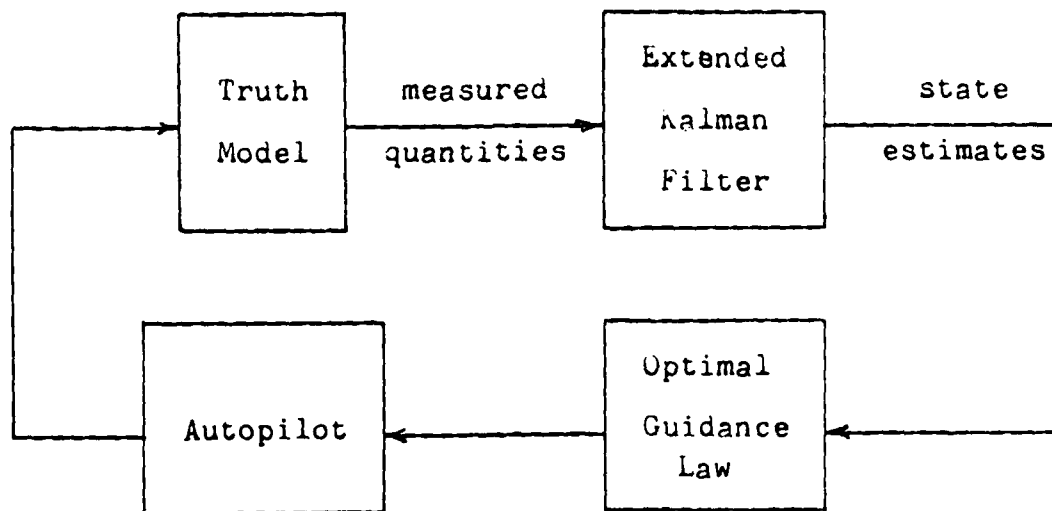

Figure 1. System Block Diagram

II. OBJECTIVES

This work, and hence its objectives, extend from findings which prompted the original summer work.[2] Certain unexplained anomalies were being observed in results obtained from the simulated system. The original work was concerned with the effects upon system performance of state inaccuracies in the nine-state extended Kalman filter. The focus of this work was toward the comparative behavior of the system under the influence of the six-state and nine-state filters. In particular, investigation was made into the fact that system performance under the influence of the nine-state filter was improved over that of the six-state filter in spite of large errors in the estimates of those states not estimated by the six-state filter.

III. MATHEMATICAL CONSIDERATIONS

A review was made of the filter models used in the simulated system. For the nine-state model (Equations (3) - (11)), no factors were brought to light which had not been noted in the earlier work. The state model used for the six-state filter, however, was found to have some inherent features which tend to explain its performance. The basic structure for the six-state filter as obtained by degenerating the state equations for the nine-state model is[3]

$$\dot{x}_1 = x_4 \tag{12}$$

$$\dot{x}_2 = x_5 \tag{13}$$

$$\dot{x}_3 = x_6 \qquad (14)$$

$$\dot{x}_4 = a_{T_x} - a_{M_x} \qquad (15)$$

$$\dot{x}_5 = a_{T_y} - a_{M_y} \qquad (16)$$

$$\dot{x}_6 = a_{T_z} - a_{M_z} \qquad (17)$$

where a_{T_x}, a_{T_y}, and a_{T_z} represent target acceleration in the x, y, and z directions, respectively. The remaining notation is the same as defined in Section I. A basic assumption was made in the formulation of this model at the very outset. Namely, the control vector $\underline{u}$ was defined as

$$\underline{u} = \begin{bmatrix} -a_{M_x} & -a_{M_y} & -a_{M_z} \end{bmatrix}^T \qquad (18)$$

and the state equations were formulated as

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \qquad (19)$$

where

$$A = \begin{bmatrix} 0 & \vdots & I \\ \cdots & \vdots & \cdots \\ 0 & \vdots & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ \cdots \\ I \end{bmatrix} \qquad (20,21)$$

and where I is a 3 by 3 identity matrix. Inherent in this formulation is the assumption that $a_{T_x} = a_{T_y} = a_{T_z} = 0$.

IV. AN EXPERIMENTAL MODEL

To exemplify the behavior of the system under the influence
of both six-state and nine-state extended Kalman filters, a
typical launch configuration was selected for simulation. For
the model selected, the aspect angle is 90°, and the boresight
angle is 0°. Simulation runs were made for launch ranges from
5000 to 13000 ft. in increments of 1000 ft. A smart target
algorithm is included in the simulation package. In this
algorithm, if range is greater than 6000 ft., target velocity
remains constant. When a range of 6000 ft. is reached, if more
than one second remains until impact, the target takes evasive
action. At one second time-to-go, the target makes a second
"last-ditch" maximum-g maneuver. Simulation results for the
system under these conditions for both six and nine-state
filters are shown in Table 1. The miss distances shown are
averages for ten Monte Carlo runs.

If the evasive target algorithm is defeated in the
simulation program to yield a target of constant velocity,
the simulation results for identical initial launch ranges
and configuration are listed in Table 2. Again, miss distances
are averages of ten Monte Carlo runs.

Table 1: Miss Distance, Evasive Target

Launch Range (ft)	Miss Distance (ft)		
	Deterministic	Six State EKF	Nine State EKF
5000	0.798	4.65	2.474
6000	2.366	9.85	3.387
7000	0.562	17.36	3.030
8000	0.475	33.71	3.024
9000	0.467	49.04	2.783
10000	0.478	57.96	3.109
11000	0.582	88.36	4.689
12000	0.621	122.54	5.103
13000	0.598	192.44	6.961

Table 2: Miss Distance, Constant-Velocity Target

Launch range (ft)	Miss Distance (ft)		
	Deterministic	Six State EKF	Nine State EKF
5000	1.586	0.947	2.425
6000	0.414	0.630	2.420
7000	0.262	0.563	2.492
8000	0.231	0.462	2.997
9000	0.217	0.613	3.403
10000	0.201	0.494	4.067
11000	0.191	0.553	5.521
12000	0.169	1.131	6.476
13000	202.44	202.26	288.15

V. DISCUSSION OF FINDINGS

The data of Table 1 shows that the nine-state filter's performance is far superior to that of the six-state filter for the evasive target, with that superiority becoming more pronounced with increasing launch range. In light of the fact that inherent in the formulation of the six-state filter (Equations (18) - (21)) is the condition that target acceleration is zero, these results are in order. The smart target algorithm presents to the six-state filter a condition which is inherently excluded. On the other hand, the nine-state filter is equipped to deal with an accelerating target, which accounts for its consistently superior performance.

Table 2 contains data for a non-accelerating target. All launch conditions are the same as for the previous case. The performance of the six-state filter is clearly improved over its performance for the evasive target. The results for the nine-state filter are approximately equivalent to those obtained with the evasive target, attesting to its capability to respond to an accelerating target. The clear superiority of the six-state filter to the nine-state filter at all ranges for this case is attributable to the fact that there is no contradiction between the state model used for the filter and the conditions encountered by the filter in the simulation. On the other hand, the nine-state filter continues with the same mismatch between its state model and that of the truth model, as treated in the

previous investigation (see Reference 2). The effect of this,
of course, is independent of target behavior; hence, it is
present in the results of both the evasive and non-evasive
targets.

An interesting phenomenon is observed in the data for the
non-evasive target for a range of 13000 ft. A sudden failure
of the system is apparent between 12000 and 13000 ft. and is
seen to be present in both filters. It cannot be attributed
to the filters, however, because the effect is also seen in the
deterministic results. What is being observed here is the
missile running out of fuel - a feature built into the simulation
program.

VI. CONCLUSIONS

When the apparent anomalies in system performance are
viewed in the light of filter modeling limitations, the per-
formances of both the six-state and nine-state filters appear
to be well within the bounds of expectation. A similar state-
ment could be made for the relative performance between the two
filters. Clearly, the six-state filter would not be satisfactory
in a realistic environment. The nine-state filter, however,
while performing quite well for the runs examined, might be
made still better through improvements in its state model.

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